APPENDIX 1

ANALYSIS OF EU SOLUTION FROM SRS TO Y-12 (1960 – 1998)

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Shipments of EU solution sent to Y-12 from 1960 through 1998 were examined. Most of the shipments occurred between 1960 and 1986 There were 1285 shipment entries over this period. One shipment with box number V3732 shipped on 01/30/967 (transfer number 223) listed 0 total and 0 ^{235}U quantities shipped. Another shipment (box number B7302 shipped on 4/29/1982, transfer number 1242) had missing values for the total quantity and total ^{235}U (kg) shipped but listed a percent enrichment for ^{235}U, ^{234}U, ^{236}U, Pu (d/m /700 U d/m), and Np (d/m/ml). These two shipments were excluded from any statistical analyses of the data.

There was a complication in analyzing the isotopic measurements. These values were not independent. As a usual rule, one laboratory measurement was used for two shipments, although this was not always the case. In determining which laboratory measurements were used over two shipments, my decision rule was that both shipments had to be made within a couple of days of each other and had to have the same isotopic value for all isotopes reported. This eliminated 502 entries over the time span leaving 781 entries with independent isotopic data.

The total quantity shipped over the 1283 shipments was 91,117.302 kg. The shipment/entries ranged from 0.002 kg to 157.190 kg. Because some of these shipments (entries) were very small, I split them into two groups. Group 1 consisted of shipments with less than 10 kg or with the percent of ^{235}U less than 5%. There were 102 shipments that fell into this category. These were mostly correction entries but there were two shipments that also fell into the duplicate measurement group for isotopics. Because group 1 also contained enrichments of less than 5%, the range in shipment size varied from 0.002 kg to 15.806 kg. The total shipped was 54.903 kg. Which is only 0.0603% of the total shipped to Y-12 over the time span. The total ^{235}U quantity shipped over these 102 entries was 27.5 kg or 0.0509% of the total. The percent enrichment varied from 4.98% to 100.00%. The amount of ^{236}U in these 102 entries was listed for only 26 entries, a total of 6.15 kg or 0.0315% of the ^{236}U total over the entire time span. Because the group 1 entries are not truly representative of the individual shipments, I believe, shipment size should be based on group 2 entries for those shipments with no available isotopic measurements.

The total uranium mass and mass for ^{235}U and ^{236}U are given in Table 2 for group 2 shipments.

Table 2: Group 2 Shipment Statistics for Total Mass, ^{235}U and ^{236}U Mass in kg

	Range	Mean	Median	St. Dev.	95% for Mean	Total	No. Entries
Total (kg)	17.87 -157.19	77.106	70.50	23.956	75.739 – 78.474	91062.399	1181
^{235}U (kg)	8.642-71.263	45.728	50.255	14.153	44.920 – 46.536	54005.273	1181
^{236}U (kg)	1.164 – 77.581	24.3785	19.703	13.086	23.470 – 25.287	19502.773	800

Group 2 has over 99% of the total material shipped and is also the group with the most isotopic measurements.

The data was analyzed using JMP® on the PC. JMP® is a SAS® product. The distribution analyses includes percentile values for 0.5%, 2.5%, 10%, 50%, 75%, 90%, and 97.5% as well as the minimum and maximum observed values. In addition, JMP gives the usual statistics (named moments in the output). These include the mean, standard deviation, standard deviation of the mean (standard deviation divided by the square root of the total number of measurements), 95% confidence bounds on the mean, total number of entries, sum of the variable, variance, skewness,

kurtosis, and the coefficient of variation (CV). The coefficient of variation is 100 x (standard deviation divided by mean). In addition, a Shapiro-Wilk W test can be done to test for normality. This test gives a value of the statistic W and the probability of getting a smaller value. W should be close to 1.0 if the data can be considered as being a sample from a normal distribution. However, if there are a lot of data points, small departures from normality will be statistically significant but not make any practical differences in estimating central tendencies (mean or median).

I looked at the total quantity shipped, quantity ^{235}U shipped, quantity ^{236}U shipped, enrichment percentages of ^{235}U, ^{234}U, and ^{236}U and the Pu and Np analyses. None of the variables had a W-test that indicated a normal population distribution. However, for several of the variables the W-statistic was greater than 0.90 indicating a normal distribution will fit the majority of the data. I also tried a lognormal fit to the data. The normal distribution fit better for all but the Pu and Np values. For these two, the lognormal distribution is a better fit. There is an additional problem for the Pu values in that some of them were reported at less than detected with the detection limit given as the upper bound. I analyzed the data using the detection limits whenever the value was less than detected. This introduces a bias into the estimates of both mean and variance but not the percentile values. The estimate of mean is an over-statement (biased high) and the variance is underestimated (biased low). I have also estimated the statistical parameters based on maximum likelihood estimates for censored data. Censored analysis means that instead of actual measured values, the values are cut-off at some point (detection limit). There were 141 measurements out of 625 total Pu measurements that were reported at less than the detection limit. Most of these were reported at a detection limit of 0.01 but there were two that were reported at a less than 0.500 detection limit. This value should be checked. The 95% confidence intervals for mean and median (50^{th} percentile) will be wider based on censored analysis than assuming all values are actual measured values rather than detection limits.

The Central Limit Theorem tells us that as the number of data points increases, the sample distribution of the mean approaches the normal distribution regardless of the individual data distribution. Thus, the normal theory can be used for estimates of central tendency (either mean or median). If the data fit a normal distribution, the mean and median are about the same. However, the mean is a more efficient estimator of the central value (has a smaller 95% confidence interval for the true population value). For the lognormal distribution, calculating the mean in the natural log space and then transforming it back to the units of the original data is the most efficient estimator of the average (50^{th} percentile). The mean of the lognormal distribution is always greater than the (Rest of appendix is in size 12) median. For the lognormal distribution the median is the best estimator for the central measure rather than the mean.

For the variables analyzed, the mean and median of the sample data are reasonably close for group 2 data values with the exception of Pu and Np. For the isotopic values, $^{235}U\%$, $^{234}U\%$, $^{236}U\%$, Pu, and Np, the differences between estimates using only group 2 entries and using all independent entries are very small. These are summarized in Table 3. Differences between estimates based on the lognormal and normal distributions for Pu and Np are given in Table 4. Also, the estimates based on the censored distribution are given in Table 4 for Pu. Only the lognormal distribution was used for the censored estimates. The empirical percentiles are the same since they are based on the data and not the assumed distribution. Recommend using the exponential of the lognormal mean as the best estimate of the central average for the distribution of Pu and Np.

The JMP® output is given in Figures 1 through 3 for the kg quantities of U, ^{235}U, and ^{236}U. Figures 4 through 10 give the group 2 analyses for the isotopics, $^{235}U\%$, $^{234}U\%$, $^{236}U\%$, Pu, and

Np for the independent measurements in group 2 only. Figures 8 and 10 give the lognormal analyses for Pu and Np, respectively. Figures 11 through 15 give the analyses for the isotopics for all independent measurements. There are 20 shipments with Pu reported in d/m/ml (10 independent measurements). These were analyzed separately. The summary of the 10 Pu measurements in d/m/ml are given in Table 5 with Figures 16 and 17 giving the JMP® output for the normal and lognormal distributions, respectively.

Table 3: Comparison of Isotopic Distributions for Group 2 versus All Independent Measurements

Variable	Statistic	Group 2	All
^{235}U%	Range	18.13 - 91.610 %	4.998 - 100.00%
	Mean	60.3521%	62.6477 %
	Median	56.520%	59.90%
	N	681	781
	St. Dev.	14.904	16.0678
	95% Conf. Int. for mean	59.226 - 61.4778%	61.519 - 63.776%
	Shapiro-Wilk W Test	0.912550	0.932303
	Prob.<W	0.0000	0.0000
^{234}U%			
	Range	0.1500 - 4.5300	0.1500 - 4.5300
	Mean	1.3958%	1.3954%
	Median	1.35%	1.35%
	N	541	544
	St. Dev.	0.2148	0.2145
	95% Conf. Int. for mean	1.3777 - 1.4140	1.3774 - 1.4135
	Shapiro-Wilk W Test	0.750313	0.751243
	Prob. <W	0.0000	0.0000
^{236}U%	Range	1.5 - 78.3%	1.5 - 78.3%
	Mean	28.2332%	27.8039%
	Median	30.510%	30.090%
	N	545	570
	St. Dev.	9.1784	9.4049
	95% Conf. Int. for mean	27.4609 - 29.0056	27.0302 - 28.5777
	Shapiro-Wilk W Test	0.952728	0.953038
	Prob. < W	0.0000	0.0000
Pu (d/m/700 U d/m)	Range	0.01 - 1.15	0.01 - 1.15
	Mean	0.0958	0.0958
	Median	0.0300	0.03
	N	625	626
	St. Dev.	0.1464	0.1463
	95% Conf. Int. for mean	0.0843 - 0.1073	0.0843 - 0.1073
	Shapiro-Wilk W Test	0.614924	0.615129
	Prob. < W	0.0000	0.0000
Np (d/m/ml)	Range	4- 2800	4 - 2800
	Mean	272.0915	271.6945
	Median	113	112
	N	634	635
	St. Dev.	413.0483	412.8436
	95% Conf. Int. for mean	239.8776 - 304.3053	239.5221 - 303.8669
	Shapiro-Wilk W Test	0.624915	0.624596
	Prob. < W	0.0000	0.0000

Table 4: Comparisons Between Estimates for Pu and Np based on Normal and Lognormal Distributions (group 2 Only)

Isotope	Median	Mean	St. Dev.	95% Conf. Int. For Mean/Median	Shapiro-Wilk W-Test (Prob < W)
Np (d/m/ml)					
Normal Distribution	113.0	272.0915	413.0483	239.8776 - 304.3053	0.624915 (0.0000)
Lognormal	113.0	284.6943	634.4645	234.2941 - 335.0854	
(Exponential of log mean)*		116.5476		105.0146 - 129.3472	0.972159 (<0.0001)
Pu (d/m/700 U d/m)					
Normal Distribution	0.03	0.0958	0.1464	0.0843 - 0.1073	0.614924 (0.0000)
Lognormal	0.03	0.0949	0.1936	0.0794 - 0.1104	0.870803 (0.0000)
(Exponential of log mean)*		0.04176		0.0378 - 0.0462	
Censored Lognormal	0.0901	0.3108		0.0652 - 0.1150	
(Exponential of censored log)*		0.0251		0.0221 - 0.0285	

*The estimate of the 50th percentile based on the lognormal distribution and the 95% confidence of the 50th percentile. For the lornormal distribution, the estimate of the median corresponds to what is generally considered the average value.

For Pu, I recommend using the estimate of 0.0251 for the average (50th percentile) with the confidence limits on the estimate of between 0.0221 and 0.0285. For Np, I recommend using 116.5476 for the estimate of the average (50th percentile) with confidence limits of 105.0146 - 129.3472. For all other isotopes, I recommend using the mean estimate based on the normal distribution.

The following describes how the lognormal means and 50th percentile estimates are determined.

Let y_i = the natural logarithm of the ith independent Pu or Np measurement.

The lognormal mean is given by

$$\bar{y} = \sum_{i=1}^{n} y_i \div n \,. \qquad (1)$$

The lognormal variance is given by

$$s_y^2 = \frac{\sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2}{\left(n-1\right)}. \qquad (2)$$

The 95% confidence intervals for the lognormal mean are given by

$$\bar{y} \pm t(0.975, n-1)s/\sqrt{n}\,, \qquad (3)$$

where t(0.975,n-1) is the 97.5th percentile of the student's t distribution with (n-1) degrees of freedom.

Transforming the lognormal mean and lognormal confidence limits back into the original data units gives an estimate of the median with confidence limits or

$$\text{Median Estimate} = \exp(\bar{y}) \text{ with confidence limits} = \exp(\bar{y} \pm t(0.975, n-1)s/\sqrt{n}). \qquad (4)$$

The mean in the original units is given by

$$\bar{X} = \exp\left(\bar{y} + S_y^2/2\right). \quad (5)$$

The variance in the original units is given by

$$S_x^2 = (\bar{X})^2\left[\exp\left(S_y^2\right)-1\right]. \quad (6)$$

The 95% confidence intervals for the mean are given by

$$\bar{X} \pm t(0.975, n-1)S_x/\sqrt{n}\ . \quad (7)$$

For censored data, define h as the proportion censored or h = c/n where c is the number of data values listed as less than the detection limit. Then compute the mean and variance estimates on the natural logarithm of the uncensored data only.

Log mean (uncensored) $\bar{Y}_c = \dfrac{\sum_{i=1}^{n-c} Y_i}{(n-c)}$. (8)

Log variance (uncensored) $S_c^2 = \dfrac{\sum_{i=1}^{n-c}\left(y_i - \bar{y}_c\right)}{(n-c)}$. (9)

Compute the ratio

$$\gamma = S_c^2/\left(\bar{y}_c - y_o\right)^2, \quad (10)$$

where $y_o = \ln(0.01)$ and 0.01 is the detection limit.

From Table A-15 in reference [1] find the value of λ corresponding to h and γ.
The lognormal mean is given by

$$\bar{Y} = \bar{Y}_c - \lambda\left(\bar{Y}_c - Y_o\right). \quad (11)$$

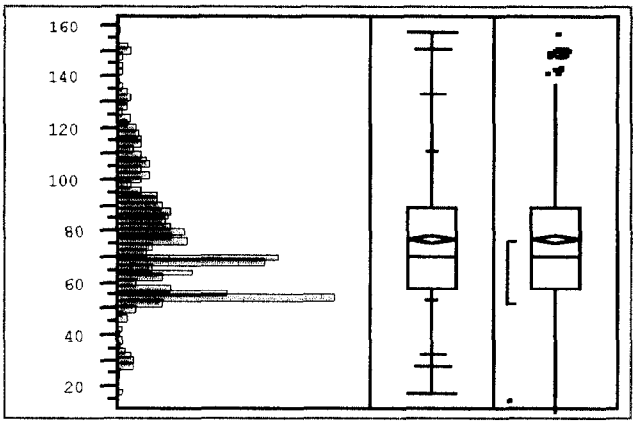
The lognormal variance is given by

$$S_y^2 = S_c^2 + \lambda\left(\bar{Y}_c - Y_o\right)^2. \quad (12)$$

The mean and median with corresponding 95% confidence intervales based on the censored estimates (Eq. (11) and Eq. (12)) are converted to the original units by substituting the values found in Eqs. (11) and (12) into Eq. (4) for the median or 50th percentile and Eqs. (5) and (7) for the mean.

Figure 1: JMP Output for Group 2 Distribution of Total Quantity Shipped (kg)

Quantities (Kg) Shipped



Quantiles (kg) Shipped

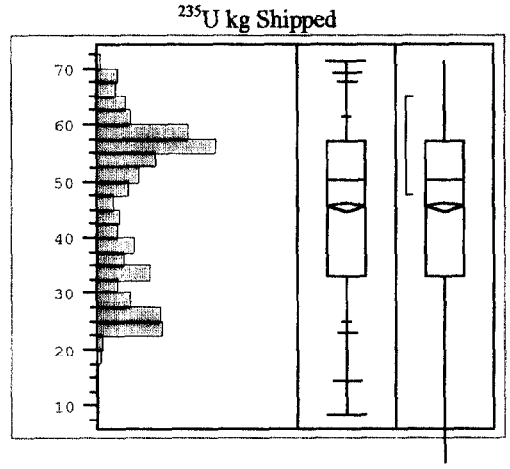
	Quantile	Kg. shipped
maximum	100.0%	157.19 kg
	99.5%	151.16 kg
	97.5%	133.43 kg
	90.0%	111.24 kg
quartile	75.0%	89.64 kg
median	50.0%	70.50 kg
quartile	25.0%	57.57 kg
	10.0%	53.63 kg
	2.5%	32.80 kg
	0.5%	28.08 kg
minimum	0.0%	17.87 kg

Moments

Mean	77.106
Std Dev	23.956
Std Error Mean	0.697
Upper 95% Mean	78.474
Lower 95% Mean	75.739
N	1181
Sum	91062.399
Variance	573.866
Skewness	0.744
Kurtosis	0.513
CV	31.068

Test for Normality
Shapiro-Wilk W Test: W = 0.939271 Prob<W = 0.0000.

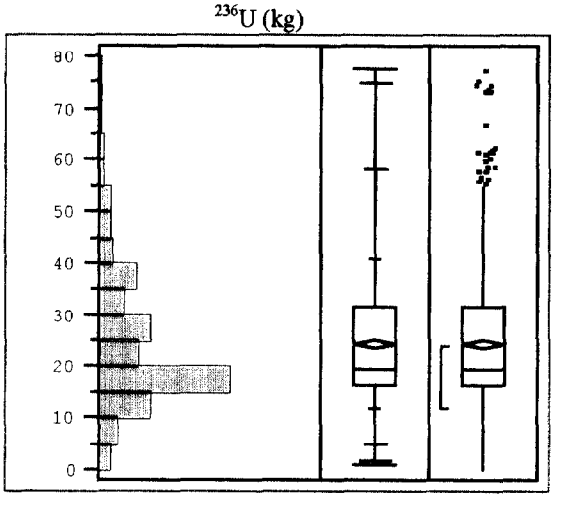
Figure 2: JMP Output for Group 2 Quantity of ^{235}U Shipped (kg).



Quantiles	Moments			
maximum	100.0%	71.263	Mean	45.728
	99.5%	69.403	Std Dev	14.153
	97.5%	67.878	Std Error Mean	0.412
	90.0%	61.539	Upper 95% Mean	46.536
quartile	75.0%	57.365	Lower 95% Mean	44.920
median	50.0%	50.255	N	1181
quartile	25.0%	33.208	Sum	54005.273
	10.0%	25.090	Variance	200.321
	2.5%	23.191	Skewness	-0.314
	0.5%	14.411	Kurtosis	-1.231
minimum	0.0%	8.642	CV	30.951

Test for Normality
Shapiro-Wilk W Test W = 0.905807 Prob<W = 0.0000

Figure 3: JMP Output for Group 2 Quantity ^{236}U Shipped (kg)

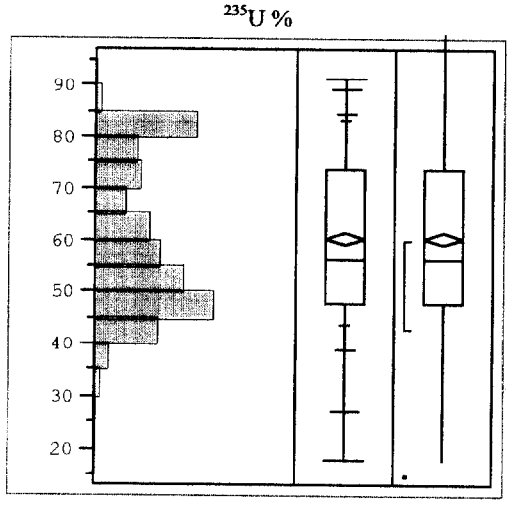


Quantiles			Moments	
	Quantile	kg. Shipped		
maximum	100.0%	77.581 kg	Mean	24.3785
	99.5%	75.053 kg	Std Dev	13.0855
	97.5%	58.423 kg	Std Error Mean	0.4626
	90.0%	40.884 kg	Upper 95% Mean	25.2866
quartile	75.0%	31.915 kg	Lower 95% Mean	23.4703
median	50.0%	19.703 kg	N	800
quartile	25.0%	16.416 kg	Sum	19502.773
	10.0%	12.001 kg	Variance	171.2292
	2.5%	5.001 kg	Skewness	1.1773
	0.5%	1.997 kg	Kurtosis	1.6706
Minimum	0.0%	1.164 kg	CV	53.6763

Test for Normality	Shapiro-Wilk W Test	
	W = 0.903769	Prob<W = 0.0000

The JMP output for the isotopic distribution based on Group 2 data is given in Figures 4 through 10 after omitting duplicate laboratory analyses. (Only the independent laboratory values were analyzed for the group 2 shipments.) The non-detected values for Pu were replaced by their associated detection limits which leads to a conservative estimate of mean but not for variance. Also 20 shipments (10 independent measurements) had Pu values in d/m/ml. These are analyzed separately. The lognormal distribution analyses for Pu and Np are given in Figures 9 and 10, respectively.

Figure 4: JMP Output for Distribution of ^{235}U % for Group 2 (Independent Measurements)

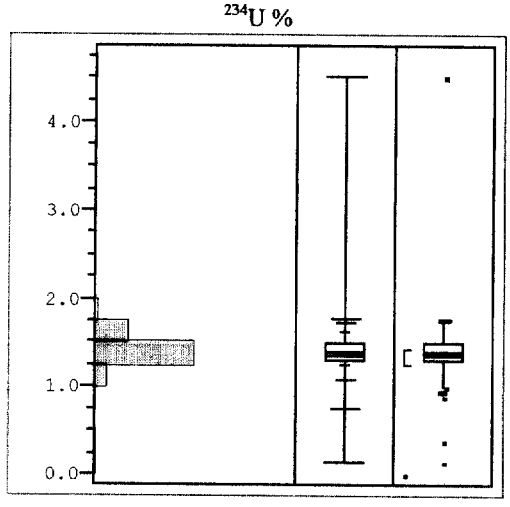


	Quantiles		Moments	
maximum	100.0%	91.610	Mean	60.3521
	99.5%	89.357	Std Dev	14.9604
	97.5%	84.640	Std Error Mean	0.5733
	90.0%	83.528	Upper 95% Mean	61.4778
quartile	75.0%	73.835	Lower 95% Mean	59.2265
median	50.0%	56.520	N	681
quartile	25.0%	48.084	Sum	41099.808
	10.0%	43.882	Variance	223.8122
	2.5%	38.956	Skewness	0.3044
	0.5%	27.158	Kurtosis	-1.0525
minimum	0.0%	18.130	CV	24.7884

Test for Normality

Shapiro-Wilk W Test W = 0.912550 Prob<W = 0.0000

Figure 5: JMP Output for Group 2 Distribution Analysis of ^{234}U % (Independent Measurements)



Quantiles

	Quantile	% ^{234}U
maximum	100.0%	4.5300
	99.5%	1.7829
	97.5%	1.7400
	90.0%	1.6380
quartile	75.0%	1.4900
median	50.0%	1.3500
quartile	25.0%	1.3000
	10.0%	1.2500
	2.5%	1.0900
	0.5%	0.7518
minimum	0.0%	0.1500

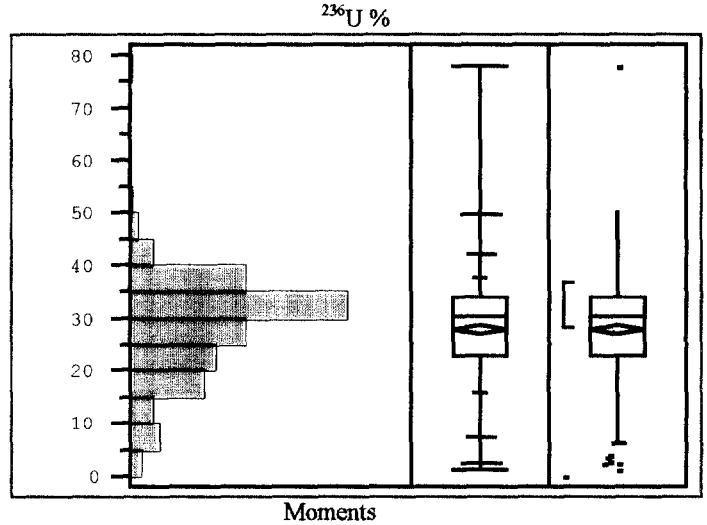
Moments

Mean	1.3958
Std Dev	0.2148
Std Error Mean	0.0092
Upper 95% Mean	1.4140
Lower 95% Mean	1.3777
N	541
Sum	755.1360
Variance	0.0461
Skewness	5.3284
Kurtosis	85.7466
CV	15.3879

Test for Normality

Shapiro-Wilk W Test	W = 0.750313	Prob<W = 0.0000

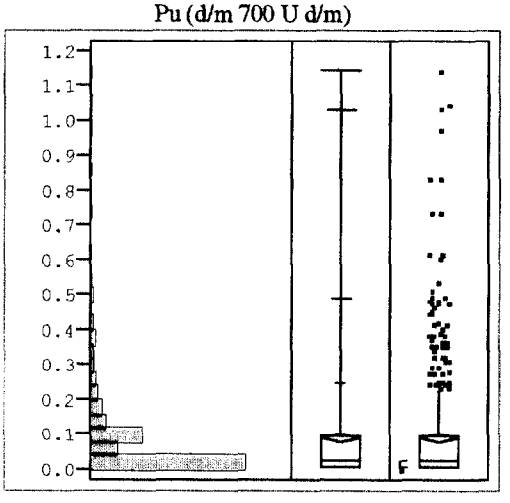
Figure 6: JMP Output for Group 2 Distribution Analysis of ^{236}U % (Independent Measurements)



Quantiles			Moments	
maximum	100.0%	78.300	Mean	28.2332
	99.5%	49.974	Std Dev	9.1784
	97.5%	42.365	Std Error Mean	0.3932
	90.0%	37.580	Upper 95% Mean	29.0056
quartile	75.0%	34.110	Lower 95% Mean	27.4609
median	50.0%	30.510	N	545
quartile	25.0%	23.170	Sum	15387.117
	10.0%	15.856	Variance	84.2435
	2.5%	7.412	Skewness	-0.3526
	0.5%	2.671	Kurtosis	1.5679
minimum	0.0%	1.500	CV	32.5093

Test for Normality
Shapiro-Wilk W Test W = 0.952788 Prob<W = 0.0000

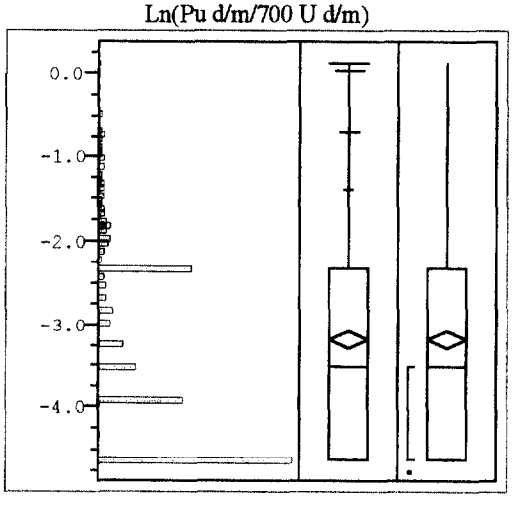
Figure 7: JMP Output for Group 2 Distribution of (Pu d/m /700 U d/m)
(Excluding 20 shipments, 10 independent Analyses reported in units of Pu (d/m/ml)



Quantiles			Moments	
maximum	100.0%	1.1500	Mean	0.0958
	99.5%	1.0322	Std Dev	0.1464
	97.5%	0.4935	Std Error Mean	0.0059
	90.0%	0.2500	Upper 95% Mean	0.1073
quartile	75.0%	0.1000	Lower 95% Mean	0.0843
median	50.0%	0.0300	N	625
quartile	25.0%	0.0100	Sum	59.8640
	10.0%	0.0100	Variance	0.0214
	2.5%	0.0100	Skewness	3.3717
	0.5%	0.0100	Kurtosis	15.0498
minimum	0.0%	0.0100	CV	152.8589

Test for Normality
Shapiro-Wilk W Test W = 0.614924 Prob<W = 0.0000

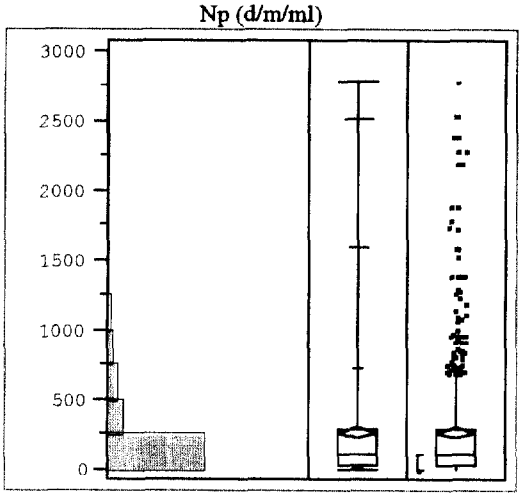
Figure 8: Lognormal Distribution of Independent Pu Measurements in Group 2.



Quantiles			Moments	
maximum	100.0%	0.1398	Mean	-3.1758
	99.5%	0.0315	Std Dev	1.2812
	97.5%	-0.7063	Std Error Mean	0.0512
	90.0%	-1.3863	Upper 95% Mean	-3.0751
quartile	75.0%	-2.3026	Lower 95% Mean	-3.2764
median	50.0%	-3.5066	N	625
quartile	25.0%	-4.6052	Sum	-1984.844
	10.0%	-4.6052	Variance	1.6414
	2.5%	-4.6052	Skewness	0.3913
	0.5%	-4.6052	Kurtosis	-1.0403
minimum	0.0%	-4.6052	CV	-40.3422

Test for Normality
Shapiro-Wilk W Test W = 0.870803 Prob<W = 0.0000

Figure 9: JMP Output for Distribution Analysis of Np (d/m/ml)



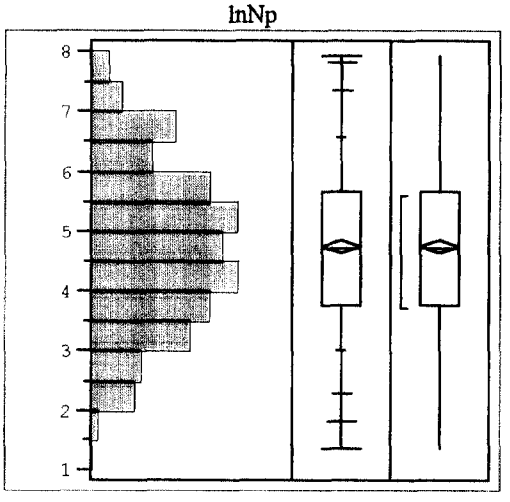
Quantiles			Moments	
maximum	100.0%	2800.0	Mean	272.0915
	99.5%	2523.8	Std Dev	413.0483
	97.5%	1600.0	Std Error Mean	16.4042
	90.0%	736.8	Upper 95% Mean	304.3053
quartile	75.0%	298.0	Lower 95% Mean	239.8776
median	50.0%	113.0	N	634
quartile	25.0%	43.0	Sum	172506
	10.0%	21.0	Variance	170608.86
	2.5%	10.0	Skewness	3.0309
	0.5%	6.2	Kurtosis	10.9735
minimum	0.0%	4.0	CV	151.8049

Test for Normality

Shapiro-Wilk W Test W = 0.62915

Prob<W = 0.0000

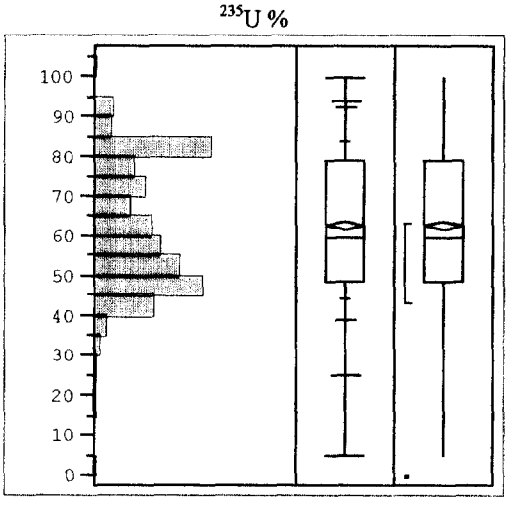
Figure 10: Lognormal Distribution for Np Group 2



Quantiles			Moments	
maximum	100.0%	7.9374	Mean	4.7583
	99.5%	7.8332	Std Dev	1.3365
	97.5%	7.3778	Std Error Mean	0.0531
	90.0%	6.6012	Upper 95% Mean	4.8625
quartile	75.0%	5.6971	Lower 95% Mean	4.6541
median	50.0%	4.7273	N	634
quartile	25.0%	3.7612	Sum	3016.7669
	10.0%	3.0445	Variance	1.7862
	2.5%	2.3026	Skewness	0.0879
	0.5%	1.8187	Kurtosis	-0.6028
minimum	0.0%	1.3863	CV	28.0876
Test for Normality				
Shapiro-Wilk W Test		W = 0.972159	Prob<W = <.0001	

The JMP output for the isotopics using all the independent data is given in Figures 11 through 15. The Pu d/m /700 U d/m values that were less than detection have been replaced with the detection limit, leading to a conservative estimate of mean but not variance. The 20 shipment values (10 independent measurements) of Pu in units d/m/ml have been analyzed separately.

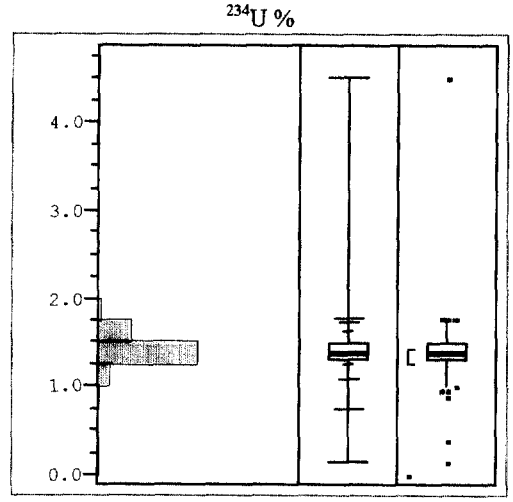
Figure 11: JMP Output for Distribution of ^{235}U % (all independent values)



Quantiles			Moments	
maximum	100.0%	100.00	Mean	62.6477
	99.5%	94.18	Std Dev	16.0678
	97.5%	92.69	Std Error Mean	0.5749
	90.0%	84.18	Upper 95% Mean	63.7763
quartile	75.0%	79.04	Lower 95% Mean	61.5190
median	50.0%	59.90	N	781
quartile	25.0%	48.69	Sum	48927.815
	10.0%	44.04	Variance	258.1729
	2.5%	39.16	Skewness	0.1637
	0.5%	24.99	Kurtosis	-0.9474
minimum	0.0%	4.998	CV	25.6478

Test for Normality
Shapiro-Wilk W Test W = 0.932303 Prob<W = 0.0000

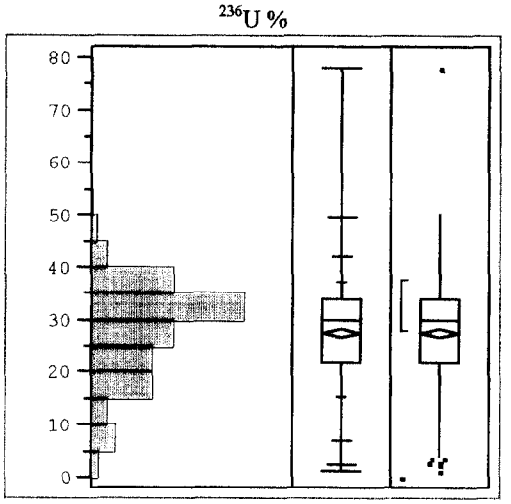
Figure 12: JMP Output for All Independent values of ^{234}U %



	Quantiles		Moments	
maximum	100.0%	4.5300	Mean	1.3954
	99.5%	1.7827	Std Dev	0.2145
	97.5%	1.7400	Std Error Mean	0.0092
	90.0%	1.6350	Upper 95% Mean	1.4135
quartile	75.0%	1.4900	Lower 95% Mean	1.3774
median	50.0%	1.3500	N	544
quartile	25.0%	1.3000	Sum	759.1180
	10.0%	1.2500	Variance	0.0460
	2.5%	1.0900	Skewness	5.3255
	0.5%	0.7595	Kurtosis	85.8054
minimum	0.0%	0.1500	CV	15.3697

Test for Normality
Shapiro-Wilk W Test W = 0.751243 Prob<W = 0.0000

Figure 13: JMP Output of Distribution Analysis of ^{236}U% (all independent Measurements)



Quantiles			Moments	
maximum	100.0%	78.300	Mean	27.8039
	99.5%	49.935	Std Dev	9.4049
	97.5%	42.334	Std Error Mean	0.3939
	90.0%	37.424	Upper 95% Mean	28.5777
quartile	75.0%	33.983	Lower 95% Mean	27.0302
median	50.0%	30.090	N	570
quartile	25.0%	22.078	Sum	15848.245
	10.0%	15.403	Variance	88.4527
	2.5%	7.031	Skewness	-0.3499
	0.5%	2.694	Kurtosis	1.2725
minimum	0.0%	1.500	CV	33.8259

Test for Normality
Shapiro-Wilk W Test W = 0.953038 Prob<W = 0.0000

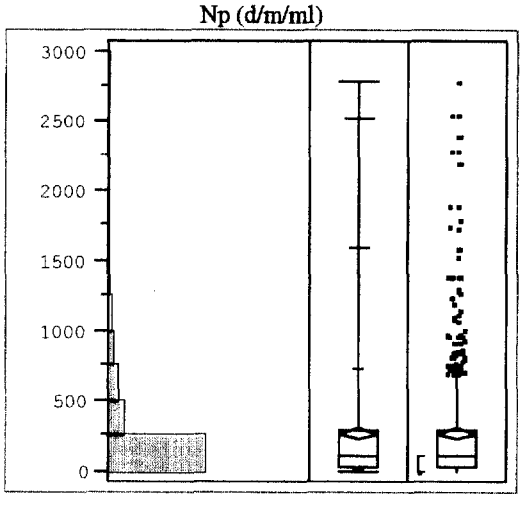
Figure 14: JMP Output of Distribution Analysis for Pu (d/m /700 U d/m/) (All independent Measurements)

Pu (d/m /700 U d/m)

	Quantiles		Moments		
maximum	100.0%	1.1500	Mean	0.0958	
	99.5%	1.0319	Std Dev	0.1463	
	97.5%	0.4932	Std Error Mean		0.0058
	90.0%	0.2500	Upper 95% Mean		0.1073
quartile	75.0%	0.1000	Lower 95% Mean		0.0843
median	50.0%	0.0300	N	626	
quartile	25.0%	0.0100	Sum	59.9740	
	10.0%	0.0100	Variance	0.0214	
	2.5%	0.0100	Skewness	3.3738	
	0.5%	0.0100	Kurtosis	15.0759	
minimum	0.0%	0.0100	CV	152.7015	

Test for Normality
Shapiro-Wilk W Test W = 0.615129 Prob. < W = 0.0000

Figure 15: JMP Output of Distribution Analysis of Np (d/m/ml) for all Indpendent Measurements



	Quantiles		Moments	
maximum	100.0%	2800.0	Mean	271.6945
	99.5%	2523.0	Std Dev	412.8436
	97.5%	1600.0	Std Error Mean	16.3832
	90.0%	735.2	Upper 95% Mean	303.8669
quartile	75.0%	298.0	Lower 95% Mean	239.5221
median	50.0%	112.0	N	635
quartile	25.0%	43.0	Sum	172526
	10.0%	21.0	Variance	170439.84
	2.5%	10.0	Skewness	3.0350
	0.5%	6.2	Kurtosis	10.9909
minimum	0.0%	4.0	CV	151.9514

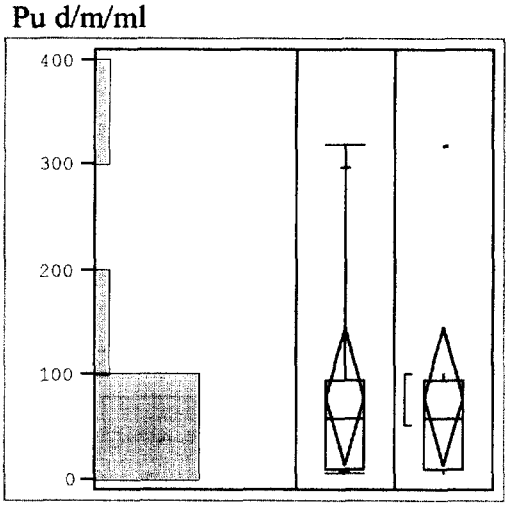
Test for Normality
Shapiro-Wilk W Test W = 0.624596 Prob<W = 0.0000

There were 20 shipments with 10 independent measurements for Pu in d/m/ml. These have also been analyzed assuming both a normal and a lognormal distribution. These values were all above detection. The results are presented in Table 5 and the output in Figures 16 and 17 for the normal and lognormal distributions, respectively.

Table 5: Pu Estimates for 20 shipments measured in d/m/ml (10 Independent Analyses)

Distribution	Range	Mean	Median	St. Dev.	95% Conf. Inter. (Mean)	Shapiro -Wilk W	Prob< W
Normal	8-320	80.5	60	90.8323	15.5220-145.4780	0.708639	0.0014
Lognormal	8-320	92.7057	60	156.0475	-18.916-204.328	0.911299	0.2774
(Exponential of log mean)		47.3498	60	20.6625-108.5065			

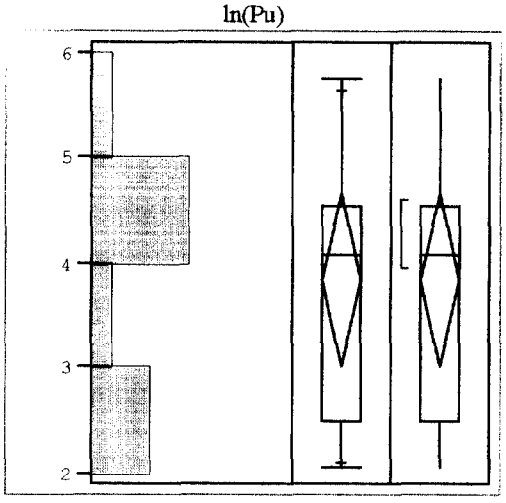
Figure 16: Normal Distribution Analyses for Pu d/m/ml



	Quantiles		Moments	
maximum	100.0%	320.00	Mean	80.5000
	99.5%	320.00	Std Dev	90.8323
	97.5%	320.00	Std Error Mean	28.7237
	90.0%	298.20	Upper 95% Mean	145.4780
quartile	75.0%	96.00	Lower 95% Mean	15.5220
median	50.0%	60.00	N	10
quartile	25.0%	12.00	Sum	805.0000
	10.0%	8.40	Variance	8250.5000
	2.5%	8.00	Skewness	2.3515
	0.5%	8.00	Kurtosis	6.4435
minimum	0.0%	8.00	CV	112.8351

Test for Normality
Shapiro-Wilk W Test W = 0.708639 Prob<W = 0.0014

Figure 17: Lognormal Distribution Analysis of Pu Measurements in d/m/ml



Quantiles			Moments	
maximum	100.0%	5.7683	Mean	3.85756
	99.5%	5.7683	Std Dev	1.15920
	97.5%	5.7683	Std Error Mean	0.36657
	90.0%	5.6540	Upper 95% Mean	4.68681
quartile	75.0%	4.5637	Lower 95% Mean	3.02832
median	50.0%	4.0943	N	10
quartile	25.0%	2.4849	Sum	38.57564
	10.0%	2.1200	Variance	1.34374
	2.5%	2.0794	Skewness	-0.25213
	0.5%	2.0794	Kurtosis	-0.57905
minimum	0.0%	2.0794	CV	30.05003

Test for Normality		
Shapiro-Wilk W Test	W = 0.911299	Prob<W = 0.2774

In addition, I have looked at the large values of Pu, those exceeding 0.500 d/m/ 700 U d/m. There were 10 of them that occurred for 18 shipments. Two of these values were reported at less than detection. This is surprising since most of the less than detection values were reported with a detection limit of 0.01 d/m /700U d/m. These might be in error. Table 6 lists the 10 large values.

Table 6 Values of Pu Greater than 0.500 d/m /700 U d/m

Date Shipped	(Pu d/m /700U d/m)	No. Shipments
12/26-26/1962	1.150	(2)
9/16-25/1971	1.050	(2)
12/24-25/1962	1.040	(2)
12/28-29/1962	0.980	(2)
08/01/1972	0.740	(1)
09/05/1972	0.740	(1)
4/13/65	0.540	(2)
2/10/1976	0.520	(2)
2/24/1976	0.500	(2)
9/13/1985	0.500	(2) Reported at less than detected.

CONCLUSIONS:

The mass shipped for U, ^{235}U, and ^{236}U should be based on group 2 analyses. The enrichment percents for ^{235}U, ^{234}U, and ^{236}U can be modeled using a normal distribution. The contaminants Pu and Np should be modeled using a lognormal distribution. The Pu in units d/m /700 U d/m/ml have censored data (values reported at detection limits) and the censored analyses for a lognormal distribution should be used. The Pu analyses in d/m/ml involve only 20 shipments (10 independent Pu measurements) and can be modeled using a lognormal distribution.

REFERENCES:

[1] Richard O. Gilbert (1987); *Statistical Methods for Environmental Pollution Monitoring*, Van Nostrand Reinhold, New York.